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Erratum

Erratum to: "Ergodicity and exponential β-mixing bound for multidimensional diffusions with jumps" [Stochastic Process. Appl. 117 (2007) 35–56]

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1. In Assumption 2(a), a bounded transition density of Y_Δ^u was supposed to exist. However, we actually needed that of X_Δ, which was used in the proof of Claim 2 (in Proposition 3.1). The previous proof holds true as it is by replacing the statement of Assumption 2(a) with the following:

Assumption 2(a)′. *For $u \in (0,1)$, define $b^u(x) = b(x) - \int_{u<|z|\le 1} \zeta(x,z)\nu(\mathrm{d}z)$, and consider the diffusion process Y^u given by $Y_t^u = x + \int_0^t b^u(Y_s^u)\mathrm{d}s + \int_0^t \sigma(Y_s^u)\mathrm{d}w_s$. Then, for every $u \in (0,1)$ there exists a constant $\Delta > 0$ such that $P_x[Y_\Delta^u \in B] > 0$ for any $x \in \mathbb{R}^d$ and any open set $B \subset \mathbb{R}^d$, and that, for every $X_0 = x \in \mathbb{R}^d$, X_Δ admits a transition density $(x,y) \mapsto p_\Delta(x,y)$ with respect to the Lebesgue measure on $\mathbb{R}^d$ fulfilling $\sup_{y\in\mathbb{R}^d}\sup_{x\in K} p_\Delta(x,y) < \infty$ for any compact $K \subset \mathbb{R}^d$.*

The first half of Assumption 2(a)′ makes the proof of Claim 1 (in Proposition 3.1) to remain the same. Thus, we actually need a little bit more than the previous assumptions.

2. In the proof of Lemma 2.4(i) we set $q \in (0,1)$ from the beginning, although the condition Lemma 2.4(i) contains q possibly lying in $[1,2)$; this point does not affect Lemma 2.4(ii), while the assertion "$\mathcal{A}f(x) \le \mathcal{G}f(x) + o(1)$" fails. We shall correct this point by modifying the condition in a simple manner. The resulting form is:

Lemma 2.4′. *Suppose that Assumption 1 holds true, that $|\sigma(x)| = o(|x|^{1-q/2})$ for $|x| \to \infty$, that $\int_{|z|>1} |z|^q \nu(\mathrm{d}z) < \infty$ for some $q \in (0,2)$, and that $\sup_{x\in\mathbb{R}^d} |\zeta(x,z)| \lesssim |z|$ for each $z \in \mathbb{R}^{r_2}$.*

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Then we have: (i) *Assumption* 3 *holds true if* $|x|^{q-2}x^\top b(x) \to -\infty$ *or* $|x|^{-1}x^\top b(x) \to -\infty$ *for every* $|x|$ *large enough, according to* $q \in (0,1)$ *or* $q \in [1,2)$*;* (ii) *Assumption* 3^* *holds true if there exists a constant* $c_1^* > 0$ *such that* $|x|^{-2}x^\top b(x) \le -c_1^*$ *for every* $|x|$ *large enough.*

Proof. Let $f : \mathbb{R}^d \to \mathbb{R}_+$ be a C^2 function such that $f(x) = |x|^q$ for $|x| \ge K > 0$, and that $f(x) \le |x|^q$ for every $x \in \mathbb{R}^d$. The previous Eq. (28) is correct as it is, so that $\mathcal{J}_* f(x) = o(1)$ for $|x| \to \infty$ for any $q \in (0,2)$. Turning to $\mathcal{J}^* f$, let us first suppose $q \in (0,1)$. Then it follows from the triangular inequality and the choice of f that $\mathcal{J}^* f(x) \le \int_{|z|>1}(|x+\zeta(x,z)|^q - |x|^q)\nu(\mathrm{d}z) \lesssim \int_{|z|>1}|z|^q\nu(\mathrm{d}z) \lesssim 1$. In the case of $q \in [1,2)$, without loss of generality we may suppose that $\sup_{x\in\mathbb{R}^d}|\nabla f(x)| \cdot |x|^{-(q-1)} < \infty$, so that Assumption 1 and Taylor's formula yield that $\mathcal{J}^* f(x) \lesssim \int_{|z|>1}|x+\zeta(x,z)|^{q-1}|\zeta(x,z)|\nu(\mathrm{d}z) \lesssim |x|^{q-1}+1$. Thus we have $\mathcal{J} f(x) = \mathcal{J}_* f(x) + \mathcal{J}^* f(x) \lesssim o(1) + 1 + |x|^{q-1}$ for $|x| \ge K$ whatever $q \in (0,2)$ is. Therefore, under the condition $|\sigma(x)| = o(|x|^{1-q/2})$, we can find constants $C_{\mathcal{J}} > 0$ and $K' \ge K$ for which $\mathcal{A}f(x) \le q|x|^{q-2}x^\top b(x) + o(1) + C_{\mathcal{J}}(1+|x|^{q-1}) = |x|^{q-1}(q|x|^{-1}x^\top b(x) + C_{\mathcal{J}}) + o(1) + C_{\mathcal{J}} = |x|^q\{q|x|^{-2}x^\top b(x) + o(1)\} + o(1)$ as soon as $|x| \ge K'$. The claims follows from the last expressions. □

Remark 1. Prior to the author, Kulik [1, Proposition 4.1] obtained a milder condition for the exponential β-mixing property in the case of $\sigma \equiv 0$.

Remark 2. Taking the specific value of $C_{\mathcal{J}}$ in the above proof into account, it is possible to give a weaker condition for Assumption 3.

Remark 3. In the proof of (10) in Theorem 2.2 in the original version we set $|x|^q \le f^*(x)$ for every $x \in \mathbb{R}^d$, differently from the proof of Lemma 2.4′(ii). This does not matter since for every $\epsilon > 0$ we can choose (without loss of generality) f so that $\sup_{x\in\mathbb{R}^d}\{|x|^q - f(x)\} < \epsilon$, leading to (10) with the same upper bound. So we can still use (10) under the conditions of case (ii) of Lemma 2.4 ′.

3. The proof of Lemma 2.5 contains some mistakes in estimating $\mathcal{J}^* f(x)$: what should be estimated from the above is not $|\mathcal{J}^* f(x)|$ but $\mathcal{J}^* f(x)$. However, Lemma 2.5(i) is correct as it is as clarified below. As in the original version, define f as $f(x) = \log(1+|x|)$ for $|x| \ge K > 0$, and additionally suppose $f(x) \le \log(1+|x|)$ for every $x \in \mathbb{R}^d$. We have already seen that $\mathcal{J}_* f(x) = o(1)$ for $|x| \to \infty$. On the other hand we have $\mathcal{J}^* f(x) \le \int_{|z|>1}\{\log(1+|x+\zeta(x,z)|) - \log(1+|x|)\}\nu(\mathrm{d}z) \le \int_{|z|>1}\log\{1+C|z|/(1+|x|)\}\nu(\mathrm{d}z)$, where $C := \sup_{x\in\mathbb{R}^d}|\zeta(x,z)|/|z|$ is finite under the assumption. By means of the Lebesgue theorem, the upper bound tends to 0 as $|x| \to \infty$ if $\int_{|z|>1}\log(1+|z|)\nu(\mathrm{d}z) < \infty$. Moreover, I warn that the condition of Lemma 2.5(ii) is never fulfilled as the function $x \mapsto |x|^{-1}(1+|x|)^{-1}x^\top b(x)$ is bounded under Assumption 1. Hence, in this very special case we do not know whether or not X can be exponentially β-mixing.

Remark 4. Building on the above argument and applying Lemma 2.4′(ii), the proof of Theorem 2.6(i) is correct as it is.

Some typos

- In the 14th line in page 49, the cited “[9, Theorem II.1.33 b)]” should be replaced with “[9, Proposition II.1.28]”.

- In the fifth line from the bottom in page 51, the undefined notation E_0 should be replaced with $\{z \in \mathbb{R}^{r_2} : |z| \leq 1\}$.
- The first sentence of the second paragraph in the proof of Proposition 4.1 should be read as follows: "Under Assumption ϕ, for any $a \in \mathbb{R}^d$ and $\epsilon > 0$ we can find nonrandom finite sequences $(t_i)_{i=0}^N \subset [0, t]$ and $(z_i)_{i=1}^N \subset \text{supp}(\nu) \setminus \{0\}$, for which the corresponding $\phi^{0,\xi}$ fulfills $\phi_t^{0,\xi} \in B(a; \epsilon/4)$".

References

[1] A. Kulik, Exponential ergodicity of the solutions to SDE's with a jump noise, Stochastic Process. Appl. 119 (2) (2009) 602–632.